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QED from six-dimensional vortex and gauge anomalies

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Abstract: Starting from an anomaly-free abelian Higgs model coupled to gravity in a 6-dimensional space-time we construct an effective four-dimensional theory of charged fermions interacting with U(1) abelian gauge field and gravity, both localised near the core of a Nielsen-Olesen vortex configuration. We show that an anomaly free theory in 6-dimensions can give rise to an anomalous theory in D=4, which suggests a possibility of consistent regularisation of abelian anomalous chiral gauge theories in four dimensions. We also show that the spectrum of charged bulk fermions has a mass gap.

Keywords: Field Theories in Higher Dimensions, Solitons Monopoles and Instantons.

Contents

1. Introduction

The idea that our observable 4-dimensional universe may be a brane extended in some higher-dimensional space-time has a long history [1]–[6] and has been the subject of many recent studies (for a review see, e.g. [7]). To implement this idea in practical terms one needs to find physical mechanisms which localise the higher-dimensional fields to a 3-dimensional brane whose world volume is our 4-dimensional space-time. D-branes of string theory [8] may provide a natural mechanism for localisation of the fields of the standard model. It is natural to ask whether local quantum field theory formulated in higher dimensional space-time can achieve this goal.

It was noticed already in [1] that this is not an easy task. Although the localisation of fermionic fields is quite easy to achieve, it is much more difficult to obtain localised massless gauge fields minimally coupled to the fermions living on a brane. A purely field-theoretical mechanism for localisation of gauge fields based on confinement was proposed in [4]; it is still not clear if it can be realised, as the higher-dimensional gauge theories leading to confinement in the bulk and its absence on a brane are not known (it may work, however, in a non-field theoretical model of confinement, [9]). Gravitational interactions in higher dimensions help to improve the situation. For example, a local string-like defect in six dimensions [10] provides localisation of the gauge field that has only gravitational interactions with the string [11]. A six-dimensional model in which the sixth coordinate is non-compact *à la* Randall-Sundrum [6] and the fifth is compact *à la* Kaluza-Klein does the same job [12].

In fact, the existence of a localised zero mode for the gauge field is not enough for construction of a lower dimensional effective theory - it is required that the modes living in the bulk interact weakly with the mode living on a brane. It appears that the spectrum of the bulk gauge modes localised by gravity is gapless [12], and, therefore, the viability of four-dimensional effective theory is questionable. The arguments that it can be valid for the abelian case were presented in [9], whereas what happens in non-abelian case is obscure.

The gauge fields discussed in [11, 12] were external to the fields forming a topological defect. In the case of an infinitely thin string, the metric solution has an SO(2) isometry group and the corresponding metric component $h_{\mu\theta}$, with θ being an angular coordinate, can play the role of the U(1) gauge field [14]. In fact, a natural field theory realisation of a string in six-dimensional space-time is the Nielsen-Olesen vortex [15]. This solution admits gravity localisation [13] and contains automatically the massless U(1) gauge field, which is a mixture of the the graviton fluctuation $h_{\mu\theta}$ and the original U(1) gauge field fluctuation A_μ field forming the Nielsen-Olesen vortex. This mode was found by direct computation in [16, 17] and by symmetry arguments in [18].

It is known that fermion interaction with topological defects leads to the existence of localised fermion zero modes [19, 20]. Thus, if fermions are added to the gravitating abelian Higgs model in six dimensions, the low energy effective field theory is expected to be the four-dimensional quantum electrodynamics with 4d gravity.

The aim of the present paper is to construct explicitly such an effective theory. Fermionic zero modes in the absence of gravity and in 4 dimensions in vortex background have been studied by Jackiw and Rossi in [20]. Extending this work to six dimensions is not entirely straightforward. For one thing, the zero mode of ref. [20] is localised with the help of a Majorana-type Yukawa interaction, the structure of which is somewhat different in $D = 6$. Secondly, unlike the flat space case, the presence of gravity, as we will see, allows the existence of fermion zero modes even in the absence of Yukawa interactions. Furthermore, the Nielsen-Olesen string without gravity does not contain any localised gauge field, so that the effective field theory contains nothing but free massless fermions [22].

We shall start from an abelian Higgs model with fermions in $D = 6$ and obtain quantum electrodynamics in 4-dimensional space-time, with fermionic and gauge wave-functions spread in transverse direction in a small region in the vicinity of the core of the vortex. We shall show that with a judicious choice of variables our zero-mode fermion equations can be reduced to a form similar, but not identical, to those of [20]. We will see that a vector-like theory in six dimensions leads to a vector-like QED in four dimensions. At the same time, if one starts from a genuine chiral, but anomaly free theory in $D = 6$, the resulting $D = 4$ QED of zero modes in general contains chiral anomalies. In other words, the six-dimensional theory can be considered as a self-consistent regularisation of an anomalous U(1) gauge theory in four dimensions. We also study the bulk gauge and fermion spectrum and show that fluctuations of the gauge field have no mass gap, but those of the charged

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fermion modes are massive. The existence of fermion mass gap makes it more plausible to consider the resulting low-energy theory as a consistent four-dimensional electrodynamics.

The paper is organized as follows. In the second section we describe the bosonic sector of the model: the background solution and the localised gauge field. In the third section we construct fermion zero modes for a model in which there is an interaction between fermions and a scalar. In the fourth section we find interaction between fermions and the localised gauge field. In section 5 we will consider localisation of fermions by gravity in the absence of Yukawa coupling and discuss anomalies. In section 6 we discuss the bulk gauge and fermion fields and show the absence of mass gap for vectors and its presence in the charged fermion spectrum. Section 7 contains our conclusions.

2. The model

2.1 The action

We start from a gravity-Maxwell system in $D = 6$, coupled to a complex scalar field Φ of charge e and two chiral fermions

$$\Psi_1 = \frac{(1+\Gamma_7)}{2}\Psi_1\,, \qquad \Psi_2 = \frac{(1-\Gamma_7)}{2}\Psi_2 \tag{2.1}$$

in 4_+ and 4_- representations of SO(1,5) with U(1)-charges e_1 and e_2. Here Γ_7 is the chirality matrix in $D = 6$. The action is

$$S = \int \mathrm{d}^6x\sqrt{-G}\Big\{\frac{1}{\kappa^2}R - \frac{1}{4}F_{MN}F^{MN} - (D_M\Phi)^\dagger D^M\Phi - \mathrm{U}(\Phi) + \\ + \sum_{i=1}^{2}\bar\Psi_i\Gamma^A E_A^M \nabla_M\Psi_i + g\bar\Psi_1\Phi\Psi_2 + \text{h.c.}\Big\}\,, \tag{2.2}$$

where $D_M\Phi = \partial_M\Phi + ieA_M\Phi$ and $\nabla_M\Psi_i = (\partial_M - \Omega_M + ie_iA_M)\Psi_i$. Here $\Omega_M = \frac{1}{2}\Omega_{M[AB]}\Sigma^{AB}$ is the spin connection which takes its value in the Lie algebra of SO(1,5) with generators $\Sigma_{AB} = \frac{1}{4}[\Gamma_A, \Gamma_B]$ which along with Γ_A are six 8×8 curved space Dirac matrices with anticommutational relation $\{\Gamma_A, \Gamma_B\}_+ = 2\eta_{AB}$, $\Gamma_7 = diag(1,-1)$. The indices M, N run from 0 to 5, indices μ, ν correspond to 4-dimensional space, and the signature of the metric is chosen to be $(-,+,\ldots,+)$. The model is free from gravitational, gauge and mixed anomalies only for $e_1^2 = e_2^2$. We choose $e_1 = -e_2$, the other option $e_1 = e_2$ is the same as the first one since 4_+ representation of SO(1,5) is equivalent to 4_+^*. The Yukawa-type coupling in (2.2), which can be taken to be real, is non-zero only for $e_1 - e_2 = e$. So, the charge assignment we take in sections 3 and 4 of the paper is $e_1 = e/2$, $e_2 = -e/2$. The general case of multiple fermionic species and arbitrary fermionic charges consistent with anomaly cancellation in $D = 6$ will be considered in section 5.

In fact, a gauge-invariant mass term

$$L_M = M\overline{\Psi_1^c}\Psi_2 + \text{h.c.}\,, \tag{2.3}$$

where $\Psi^c = C\bar\Psi^T$, and C is the matrix of charge conjugation, can be added to the action (2.2). We shall not include it for the analysis of the zero mode structure, but will comment on its influence on effective field theory at the end of the section 3. This term breaks the fermionic number conservation and can be forbidden if the fermion number conservation is imposed.

The chiral spinors Ψ_1 and Ψ_2 can be unified in a single eight-component spinor as $\Psi = \Psi_1 + \Psi_2$, which does not have a well defined U(1) charge unless $e_1 = e_2$, and the fermionic part of the lagrangian (2.2) can be written in another form,

$$L_F = \bar{\Psi}\Gamma^A E_A^M(\partial_M - \Omega_M + ie_f\Gamma_7 A_M)\Psi + g\bar{\Psi}\frac{(1-\Gamma_7)}{2}\Psi\Phi + \text{h.c.} \tag{2.4}$$

The mass term (2.3) in these notations is simply

$$L_M = M\overline{\Psi^c}\frac{(1-\Gamma_7)}{2}\Psi + \text{h.c.} \tag{2.5}$$

To get another form of this lagrangian, which makes its vector-like structure and thereby the absence of anomalies evident, one can introduce a genuine Dirac spinor in six dimensions $\Psi_D = \Psi_1 + \Psi_2^c$. In these notations, the fermionic part of the lagrangian is:

$$L_F = \bar{\Psi}_D\Gamma^A E_A^M(\partial_M - \Omega_M + ie_f A_M)\Psi_D + g\bar{\Psi}_D^c\Gamma_7\Psi_D\Phi + M\bar{\Psi}_D\Psi_D + \text{h.c.} \tag{2.6}$$

In the paper we are going to use the form (2.2)–(2.5), as it allows a simpler treatment of fermionic zero modes.

2.2 Vortex solution

It has been shown in [13] that the bosonic equations derived from (2.2) admit a Nielsen-Olesen vortex-type solution for which the various field configurations are

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{e}^{A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + \mathrm{e}^{B(r)}a^2\mathrm{d}\theta^2\,, \\ \Phi &= f(r)\mathrm{e}^{in\theta}\,, \qquad aeA_\theta \;=\; (P(r)-n)\mathrm{d}\theta\,, \end{aligned} \tag{2.7}$$

where $\eta_{\mu\nu}$ is the flat metric, a is the radius of the circle covered by the coordinate $\theta \in [0, 2\pi)$. The integer n is the vortex number and, as in the flat space, the functions $f(r)$ and $P(r)$ satisfy the boundary conditions

$$\begin{aligned} f(0) &= 0\,, \qquad f(\infty) = f_0 \neq 0\,, \\ P(0) &= n\,, \qquad P(\infty) = 0\,. \end{aligned} \tag{2.8}$$

As $r \to \infty$, Φ approaches a minimum of the potential $\mathrm{U}(\Phi)$ in (2.2). The boundary conditions on the metrical functions $A(r)$ and $B(r)$ introduced in (2.7) are

$$\begin{aligned} A(0) &= 1\,, \qquad B(r\to 0) = 2\ln\frac{r}{a}\,, \\ A(r\to\infty) &= B(r\to\infty) = -2cr\,, \qquad c>0\,. \end{aligned} \tag{2.9}$$

The parameters c and a are the combinations of the Newton constant κ in the bulk, the $D = 6$ cosmological constant (related to the value of the scalar potential $\mathrm{U}(\Phi)$ at the minimum), and of the parameters of the abelian Higgs model [10, 13].

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As $r \to 0$, we recover the flat space geometry at the core of the vortex. Away from the core, the geometry is curved. In particular at $r \to \infty$, the metric does not become flat Minkowski and is in fact the ADS space. It has been shown in [10, 13] that this configuration localises the gravitational fluctuations to the 4-dimensional subspace spanned by x^μ at the core of the vortex.

2.3 Localisation of gauge fields

In [18] we made a detailed analysis of the spectrum of fields of spins 0, 1 and 2 in a warped geometry and on non-trivial gauge and scalar field backgrounds. In particular, we have given a specific mixture of the fluctuation of the vector potential and the $\theta\mu$ component of the metric which is massless in $D = 4$ and has a normalisable action. This configuration, derived from a symmetry argument similar to the one given in [21] for the SU(2) case, is given by

$$V_\mu = \frac{1}{ae} P(r) W_\mu(x, r) \,, \qquad h_{\mu\theta} = \mathrm{e}^{B(r)} W_\mu(x, r) \,, \tag{2.10}$$

where W_μ is a function of x^μ and r. Substitution of (2.10) in the spin-1 part of the quadratic action, given by equations (38) to (41) of [18], gives rise to an effective action for the vector W_μ field:

$$S(W) = -\frac{1}{2a^2}\frac{2\pi a}{e^2}\int_0^\infty \mathrm{d}r\, \mathrm{e}^{\frac{1}{2}B}\left(P^2(r) + \frac{a^2 e^2}{\kappa^2}\mathrm{e}^B\right)\int \mathrm{d}^4x\,\left[(\partial_\mu W_\nu)^2 + e^{-A}\partial_r W_\mu \partial_r W_\mu\right] . \tag{2.11}$$

The r independent W_μ corresponds to localised massless vector fields. Their effective action is

$$S(W) = -\frac{1}{2q^2}\int \mathrm{d}^4x (\partial_\mu W_\nu)^2 \,. \tag{2.12}$$

where the four-dimensional gauge coupling is

$$\frac{1}{q^2} = \frac{2\pi a}{e^2}\int_0^\infty \mathrm{d}r\, \mathrm{e}^{\frac{1}{2}B}\left(P^2(r) + \frac{a^2 e^2}{\kappa^2}\mathrm{e}^B\right) . \tag{2.13}$$

With our boundary conditions on $B(r)$ and $P(r)$, the integral over r converges at both ends. A physical interpretation of (2.12), (2.13) is that the fluctuation energy of the μ-component of the 6-dimensional Maxwell field and the Kaluza-Klein vector field $h_{\mu\theta}$ are localised to a region $r \sim max(1/c, 1/M_W, 1/M_H)$ near the core of the vortex, where M_W and M_H are the bulk vector and scalar masses in the absence of gravity and c is the parameter of the solution, defined in (2.9). The effective localised field can be taken to be

$$\left(P^2(r) + \frac{a^2 e^2}{n^2}\mathrm{e}^B\right)^{\frac{1}{2}} W_\mu(x) \,,$$

which approaches $W_\mu(x)$ as $r \to 0$ and vanishes rapidly as $r \to \infty$. It is important to note that the gravitational field plays a key role in ensuring the normalisability of the effective action in (2.10), through the factor of $\mathrm{e}^{\frac{1}{2}B}$.

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3. Fermions

3.1 Dirac equation in string background

As a first step in construction of effective field theory for fermions one has to find fermionic zero modes in the string background. This problem has been solved for a Nielsen-Olesen string in 4-dimensions in the absence of gravity in [20]. In six dimensions and without gravity similar consideration for supersymmetric abelian Higgs model has been carried out in [22]; in [23, 24] fermionic modes were considered for a global string, without gravity and gauge fields. In [25] gravity has been taken into account, but interaction of fermions with gauge and Higgs was not included. In this section we will find fermionic zero modes on a six-dimensional string incorporating gravity as well as gauge-Higgs interactions, a special cases of the general problem which was addressed in [26].

General equations for the fermions in the warped metric and in the presence of gauge and Higgs fields can be found, e.g. in [26]. Applied to our case, the Dirac equation reads:

$$\Big\{ e^{-A/2}\Gamma^{\underline{\mu}}\partial_\mu + \Gamma^{\underline{r}}\left(\partial_r + A' + \frac{B'}{4}\right) + e^{-\frac{B}{2}}\Gamma^{\underline{\theta}}\left(\frac{1}{a}\partial_\theta + i\frac{e}{2}\Gamma_7 A_\theta\right) + \tag{3.1}$$

$$+g\frac{(1-\Gamma_7)}{2}\Phi + g\frac{(1+\Gamma_7)}{2}\Phi^*\Big\}\Psi = 0\,, \tag{3.2}$$

where prime denotes the derivative with respect to r, and underlined indices correspond to flat space coordinates. Here we take $g \neq 0$ and $e_1 = -e_2 = e/2$.

A special care should be taken on the boundary conditions of fermionic wave function with respect to θ. Because of the veilbein transformation properties it has to be *antiperiodic*, $\Psi(\theta) = -\Psi(\theta+2\pi)$. This corresponds to a single valued spinor in cartesian coordinate system near $r \to 0$.

To proceed, we fix the $D = 6$ Dirac matrices in the following form: $\Gamma_{\underline{\mu}} = \gamma_\mu \times \sigma_1$, $\Gamma_{\underline{r}} = \gamma_5 \times \sigma_1$ and $\Gamma_{\underline{\theta}} = 1 \times \sigma_2$, where σ_1 and σ_2 are Pauli matrices and γ_μ are 4×4 Dirac matrices chosen in such a way that γ_5 is diagonal, i.e. $\gamma_5 = diag(1, -1)$. Taking into account that

$$\Gamma_{\underline{r\theta}} \equiv \Gamma_{\underline{r}}\Gamma_{\underline{\theta}} = \begin{pmatrix} i\gamma_5 & 0 \\ 0 & -i\gamma_5 \end{pmatrix}, \tag{3.3}$$

it is convenient to introduce a new field ψ through the definition,

$$\Psi = \exp\left\{\frac{\theta}{2}\Gamma_{\underline{r\theta}} - A - \frac{1}{4}B + \int^r \mathrm{d}\rho \mathrm{e}^{-B(\rho)/2}\left(\frac{1}{2a} - \mathrm{i}\Gamma_7\frac{e}{2}\Gamma_{\underline{r\theta}}A_\theta(\rho)\right)\right\}\psi\,, \tag{3.4}$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \tag{3.5}$$

Note that due to the prefactor $\exp\left(\frac{\theta}{2}\Gamma_{r\theta}\right)$ and the antiperiodicity of Ψ the new field ψ must be *periodic* in θ.

Substitution of eq. (3.4) to (3.1) removes the gauge field from the equations and yields the equations for 4d zero modes $\gamma^\mu\partial_\mu\psi_i = 0$:

$$
\begin{aligned}
\mathrm{e}^{+\mathrm{i}\theta\gamma_5}\left(\partial_r + \frac{\mathrm{i}}{a}\gamma_5 \mathrm{e}^{-B/2}\partial_\theta\right)\psi_1 + g\Phi\gamma_5\psi_2 &= 0\,, \\
\mathrm{e}^{-\mathrm{i}\theta\gamma_5}\left(\partial_r - \frac{\mathrm{i}}{a}\gamma_5 \mathrm{e}^{-B/2}\partial_\theta\right)\psi_2 + g\Phi^*\gamma_5\psi_1 &= 0\,.
\end{aligned} \tag{3.6}
$$

To understand better their chiral structure, we write ψ_i in terms of $D = 4$ left and right handed spinors,

$$
\psi_i = \begin{pmatrix} \psi_i^L \\ \psi_i^R \end{pmatrix}, \tag{3.7}
$$

and get the system of four equations for the four different chiral components. For the left spinors we have

$$
\begin{aligned}
\mathrm{e}^{+\mathrm{i}\theta}\left(\partial_r + \frac{\mathrm{i}}{a}\mathrm{e}^{-B/2}\partial_\theta\right)\psi_1^L + g\Phi\;\psi_2^L &= 0\,, \\
\mathrm{e}^{-\mathrm{i}\theta}\left(\partial_r - \frac{\mathrm{i}}{a}\mathrm{e}^{-B/2}\partial_\theta\right)\psi_2^L + g\Phi^*\psi_1^L &= 0\,,
\end{aligned} \tag{3.8}
$$

and, for the right spinors correspondingly

$$
\begin{aligned}
\mathrm{e}^{-\mathrm{i}\theta}\left(\partial_r - \frac{\mathrm{i}}{a}\mathrm{e}^{-B/2}\partial_\theta\right)\psi_1^R - g\Phi\;\psi_2^R &= 0\,, \\
\mathrm{e}^{+\mathrm{i}\theta}\left(\partial_r + \frac{\mathrm{i}}{a}\mathrm{e}^{-B/2}\partial_\theta\right)\psi_2^R - g\Phi^*\psi_1^R &= 0\,.
\end{aligned} \tag{3.9}
$$

These equations are similar but not identical to those of [20].

3.2 Localised fermion zero modes

The four-dimensional effective lagrangian for fermion reads

$$
\int dr d\theta\sqrt{-G}\sum_{i=1}^{2}\bar\Psi_i\Gamma^A E_A^M\nabla_M\Psi_i = \int dr d\theta \sum_{i=1}^{2}\left\{N_L(r)\bar\psi_i^L\gamma^\mu\partial_\mu\psi_i^L + N_R(r)\bar\psi_i^R\gamma^\mu\partial_\mu\psi_i^R\right\}, \tag{3.10}
$$

where

$$
\begin{aligned}
N_L(r) &= \exp\left[-\frac{A}{2} + \int^r \mathrm{d}\rho \mathrm{e}^{-B(\rho)/2}\left(\frac{1}{a} + eA_\theta(\rho)\right)\right], \\
N_R(r) &= \exp\left[-\frac{A}{2} + \int^r \mathrm{d}\rho \mathrm{e}^{-B(\rho)/2}\left(\frac{1}{a} - eA_\theta(\rho)\right)\right].
\end{aligned} \tag{3.11}
$$

This fixes the condition of normalisation of fermion zero modes. The behaviour of functions $N_{L,R}$ at zero and infinity are as follows:

$$
\begin{aligned}
N_{L,R}(r\to 0) &\to r\,, \\
N_L(r\to\infty) &\to \exp\left(cr + \frac{1-n}{ac}e^{cr}\right), \\
N_R(r\to\infty) &\to \exp\left(cr + \frac{1+n}{ac}e^{cr}\right).
\end{aligned} \tag{3.12}
$$

In the following, we take the string winding number n to be positive.

The θ dependence can be removed by the substitutions

$$\psi_1^L = e^{im\theta} u_m^L(r) \chi_m^L(x^\mu) \qquad , \psi_2^L = e^{i(m+1-n)\theta} v_m^L(r) \chi_m^L(x^\mu) \,, \tag{3.13}$$

and

$$\psi_2^R = \; e^{im\theta} u_m^R(r) \chi_m^R(x^\mu) \,, \qquad \psi_1^R = -e^{i(m+1+n)\theta} v_m^R(r) \chi_m^R(x^\mu) \,, \tag{3.14}$$

where $\chi_m^{L,R}(x^\mu)$ are two-component SO(1,3) spinors. The radial wave functions can be found from

$$\begin{aligned} \left(\partial_r - \frac{m}{a} e^{-B/2}\right) u_m^L + g f v_m^L &= 0 \,, \\ \left(\partial_r - \frac{n-1-m}{a} e^{-B/2}\right) v_m^L + g f u_m^L &= 0 \,. \end{aligned} \tag{3.15}$$

for the $D = 4$ left fermions and from

$$\begin{aligned} \left(\partial_r - \frac{m}{a} e^{-B/2}\right) u_m^R + g f v_m^R &= 0 \,, \\ \left(\partial_r + \frac{n+1+m}{a} e^{-B/2}\right) v_m^R + g f u_m^R &= 0 \,. \end{aligned} \tag{3.16}$$

for the $D = 4$ right fermions. These equations reduce exactly to the ones of refs. [20, 23] for the flat case, when $B = 2log(r/a)$. They have the symmetry $m \leftrightarrow n - 1 - m$, $u \leftrightarrow v$ for the left fermions and the symmetry $m \leftrightarrow -n - 1 - m$, $u \leftrightarrow v$ for the right ones. This property will be used later to combine chiral fields to $D = 4$ Dirac fermions.

The behaviour of the modes at small r follows immediately from [20]: solutions for left-handed fermions are always regular provided m takes integer values $m = 0, \dots, n-1$, whereas one of the independent solutions for right-handed modes is always singular at the origin. So, we concentrate on large r limit, where $B \to 2cr$ and $gf \to M_f$, where M_f is the fermion mass in the bulk.

Consider first left-handed modes. Introducing $x = e^{cr}/ac$, $m_f = M_f/c > 0$, and $u_L = x^{m_f} e^{mx} y(x)$, equation for $y(x)$ is the one for a degenerate hypergeometric function,

$$x y'' + (1 + 2m_f - (n - 2m - 1)x)\, y' - m_f (n - 2m - 1) y = 0 \,. \tag{3.17}$$

If $m = \frac{n-1}{2}$ which is possible for odd n, the solution is simply

$$y = C_1 + C_2 x^{-2m_f} \,. \tag{3.18}$$

The normalisability condition gives $C_1 = 0$ and requires $m_f \neq 0$. This solution corresponds to a left-handed chiral fermion localised on a string. If $m_f = 0$, the mode is not normalisable, since the normalisation integral diverges as $exp(cr)$.[1]

For $m \neq \frac{n-1}{2}$ the $x \to \infty$ asymptotics of the general solution to (3.17) is:

$$y = C_1 x^{-m_f} + C_2 e^{(n-1-2m)x} x^{-m_f - 1} \,, \tag{3.19}$$

[1]In examining the normalisability of a solution we need to include the contribution of the r- dependent prefactor in eq. (3.11), (3.12).

what gives for u_L

$$u_L \to C_1 e^{mx} + C_2 e^{(n-1-m)x} x^{-1} . \tag{3.20}$$

From here and (3.12) it is clear that the left-handed fermionic modes with $m = 0, \dots, n-1$ are always normalisable: to get a normalisable left mode one should take C_2=0 for $m < \frac{n-1}{2}$ and C_1=0 for $m > \frac{n-1}{2}$.

For the analysis of the right-handed modes it is sufficient to change n to $-n$ in eqs. (3.17)–(3.20). These modes are not normalisable at any choice of m.

Finally, the effective lagrangian is

$$L = \sum_{m=0}^{n-1} N_m \bar{\chi}_m^L \gamma^\mu \partial_\mu \chi_m^L , \tag{3.21}$$

where

$$N_m = 2\pi \int dr N_L(r) \left(|u_m|^2 + |v_m|^2 \right) . \tag{3.22}$$

To summarise: for $g \neq 0$ we have exactly n left-handed normalisable fermionic zero modes. So far we have assumed that n is positive. If n is negative we get the same pattern for the localised right-handed modes.

The inclusion of the bulk Majorana-type mass (2.3) adds the following term to the four-dimensional effective lagrangian:

$$M \overline{\psi_{1L}^c} \psi_{2L} + \text{h.c.} \tag{3.23}$$

In this case, the modes with number m can be put together with charge-conjugated modes $(n-1-m)$ to form a massive Dirac spinor, whereas the mode $m = \frac{n-1}{2}$ is a Majorana fermion (for n odd). As we will see in section 6, the spectrum of *charged* bulk fermions develops a mass gap, and, therefore, four-dimensional massive charged spinors are the genuine localised states. On the contrary, for neutral Majorana fermions the mass gap is absent (see section 6 and also [27]), and it represents in fact a metastable state.

4. U(1) charges in $D = 4$

Our aim in this section is to define the interaction between the localised fermionic modes and the gauge field $W_\mu(x)$, defined in (2.10). While the interaction of a part of W_μ coming from the initial gauge field V_μ can be written down immediately, the gravity part, related to the component of the metric $h_{\mu\theta}$ requires some care. To find the coupling of the "graviphoton" to ψ, one can calculate the spin connections Ω_M starting from the metric

$$ds^2 = e^{A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{B(r)} \left(a d\theta + W_\mu(x) dx^\mu \right)^2 . \tag{4.1}$$

Using standard formulae we can calculate the components of Ω_A. They are given more compactly in terms of the frame components $\Omega_A = E_A{}^M \Omega_M$. We choose the frames

$$\begin{aligned} E_a{}^\mu &= e^{-A/2} \delta_a{}^\mu , \qquad & E_\mu^\theta &= -e^{-A/2} W_\mu , \\ E_{\underline{r}}^r &= 1 , \qquad & E_{\underline{\theta}}^\theta &= e^{-B/2} , \end{aligned} \tag{4.2}$$

where $a = 0, 1, 2, 3$ and an underlined character refers to the orthonormal frame. The non-vanishing components are

$$\Omega_a = \frac{1}{4}A'\Gamma_{\underline{ra}} - \frac{1}{4}e^{B/2}W_{ab}\Gamma^{b\underline{\theta}}\,,$$
$$\Omega_{\underline{\theta}} = \frac{1}{4}B'\Gamma_{\underline{r\theta}} - \frac{1}{4}e^{B/2}W_{ab}\Sigma^{ab}\,, \tag{4.3}$$

where $W_{ab} = e^{-A}\delta_a{}^{\mu}\delta_b{}^{\nu}\,(\partial_\mu W_\nu - \partial_\nu W_\mu)$. The W_{ab} containing terms could give rise to the tree level magnetic moment couplings in 4–dimensions. In our model, due to the $D = 6$ chirality, they drop out.

Upon substitution from (3.4), (4.1) and (4.2) in (2.2) we obtain the action,

$$S_F = \int d^6x \left[N_{\varepsilon_1}(r)\bar{\psi}_1\gamma^\mu\left(\partial_\mu + \frac{i}{a}W_\mu Q_1\right)\psi_1 + N_{\varepsilon_2}(r)\bar{\psi}_2\gamma^\mu\left(\partial_\mu + \frac{i}{a}W_\mu Q_2\right)\psi_2\right], \tag{4.4}$$

where $\varepsilon_i^2 = 1$ define the chirality, i.e. $\gamma_5\psi_i = \varepsilon_i\psi_i$ with $N_{+1} = N_L$, $N_{-1} = N_R$, and $N_{L,R}$ are defined in (3.11). The U(1) charge operator acting on ψ_1 is defined by

$$Q_1 = i\partial_\theta - \frac{\varepsilon_1}{2} + \frac{n}{2} \tag{4.5}$$

and the one acting on ψ_2 is given by

$$Q_2 = i\partial_\theta + \frac{\varepsilon_2}{2} + \frac{n}{2}\,. \tag{4.6}$$

Written in terms of χ_L introduced in (3.13) the trilinear part of the effective lagrangian contain χ_L and W_μ becomes

$$L_{\rm int} = \frac{i}{a}\sum_{m=0}^{n-1} Q_m\bar{\chi}_m^L\gamma^\mu\chi_m^L W_\mu\,, \tag{4.7}$$

where

$$Q_m = 2\pi\left(m - \frac{n-1}{2}\right)\int dr N_L(r)\left(|u_m|^2 + |v_m|^2\right). \tag{4.8}$$

It is essential, that the part of the localised U(1) gauge field, coming from V_μ and proportional to $P(r)$, cancels out. Using (3.21), (3.22), (4.4) and (4.5) the $D = 4$ effective lagrangian becomes

$$L = \sum_{m=0}^{m=n-1} N_m\bar{\chi}_m\gamma^\mu\left(\partial_\mu + \frac{i}{a}q_m W_\mu\right)\chi_m\,. \tag{4.9}$$

Therefore, the four-dimensional charges of fermions are simply

$$q_m = \left(m - \frac{n-1}{2}\right)q\,, \tag{4.10}$$

where q is defined in (2.13). This means that the chiral fermion with $m = \frac{n-1}{2}$ (for odd n) is neutral. We also notice that χ_m can be unified with χ^c_{n-1-m} to form a Dirac spinor.

Thus, for the even vortex number $n = 2k$, we find k Dirac fermions with charges $\frac{1}{2}, \ldots, k-\frac{1}{2}$, whereas for an odd vortex number $n = 2k+1$ one gets k charged Dirac fermions and one neutral Weyl fermion. Such a Weyl fermion would give rise to a gravitational anomaly in two, six or ten dimensions, but has an anomaly free coupling to gravity in a four-dimensional space-time. As a result, a vector-like theory in $D = 6$ leads to a vector-like QED in $D = 4$. It is interesting to note that for even n the charges are quantised in units of half-odd integers, while for odd n we obtain integrally quantised charges.

5. Gravitational localisation of fermions and anomalies

If there is no interaction between fermions and the scalar, the values of $D = 6$ left and right handed fermionic charges can be arbitrary, up to the requirement of anomaly cancellation in six dimensions. If we have n_L left fermions with charges e^i_L and n_R right-handed fermions with charges e^i_R, the absence of gravitational anomalies requires $n_L = n_R \equiv n_F$, the absence of gauge anomalies gives $\sum(e^i_L)^4 = \sum(e^i_R)^4$, and the absence of mixed anomalies leads to $\sum(e^i_L)^2 = \sum(e^i_R)^2$ [28]. For $n_F = 1, 2$ these conditions lead necessarily to a vector-like theory in $D = 6$, whereas for $n_F \geq 3$ a genuine chiral gauge theory is allowed.[2]

To analyse the general case, it is sufficient to put the Yukawa coupling $g = 0$ in all equations of the previous sections and remove the constraint relating the charges of the fermions and the scalar. Without loss of generality all charges can be assumed to be positive, as the fermion with negative charge can be transformed into a fermion with the positive charge by operation of charge conjugation which, due to the pseudo complexity of the chiral spinor representation of SO(1, 5), commutes with chirality in $D = 6$. We take, as usual, the winding number $n > 0$. Since the results are obvious from trivial modifications of eqs. (3.10)–(3.15), we omit the details.

For each left-handed (right-handed) six-dimensional spinor with charge e^i_L (e^i_R) one has *left-handed* four-dimensional Weyl spinors with normalisable (accounting for the factor (3.12)) wave-functions

$$\psi = \exp(\pm im\theta) \exp\left[\frac{m}{a}\int^r \mathrm{d}\rho \mathrm{e}^{-B(\rho)/2}\right]\chi(x)\,, \tag{5.1}$$

where, m in the exponent stands for m_L or m_R which assume integer values in the range $m_R = 0, 1, 2\ldots < \frac{e^i_R}{e}n - \frac{1}{2}$ and $m_L = 0, 1, 2\ldots < \frac{e^i_L}{e}n - \frac{1}{2}$. Also the plus (minus) sign in the exponent refers to the left (right) $D = 6$ spinor chirality. The $D = 4$ electric charges of localised spinors can be read by acting the charge operators Q_1 or Q_2 given above in (4.5, 4.6) on ψ. The result is:

$$\frac{1}{2} + m_L - n\frac{e^i_L}{e} \quad \text{and} \quad -\frac{1}{2} - m_R + n\frac{e^i_R}{e}\,. \tag{5.2}$$

The region where $D = 4$ fermions are localised is related to c and is of the order of $r \sim \frac{1}{c}$.

For $n_F = 1, 2$ the effective 4D theory is necessary vectorlike, but for $n_F \geq 3$ the resulting theory can be chiral and anomalous. As an example, consider the string with winding number $n = 1$ and the following anomaly-free in $D = 6$ charge assignment: $e^L_1 =$

[2] We are not assuming that the values of the e's are necessarily quantized. For integer charge ratios e^i_L/e and e^i_R/e one can show that an anomaly free theory in $D = 6$ leads to anomaly free theory in $D = 4$.

0, $e_2^L = 1, e_3^L = 1$ and $e_1^R = 2/\sqrt{3}$, $e_2^R = 1/\sqrt{3}, e_3^R = 1/\sqrt{3}$, $e = 1$. Normalisable solutions do not exist for the $D = 6$ fermion with $e_1^L = 0$, while for other fermions only the choice of $m_L = m_R = 0$ leads to localised $D = 4$ fermions. So, in $D = 4$ we have five left-handed fermions with charges $-1/2$, $-1/2$, $(2/\sqrt{3} - 1/2)$, $(1/\sqrt{3} - 1/2)$ and $(1/\sqrt{3} - 1/2)$. The four-dimensional anomaly cancellation equation $\sum (q_i^L)^3 = 0$ is not satisfied. The fact that anomalous U(1) gauge theory in four dimensions can be constructed as an effective theory out of an anomaly free theory in higher dimensions means that an abelian anomalous gauge theory can be made mathematically consistent. Indeed, there has been some attempts in the past (see, e.g. [29] and references therein) to make physical sense out of anomalous chiral gauge theories. In our scheme the $D = 4$ theory is a low energy approximation to a bigger and consistent theory in $D = 6$. In this sense our construction can be considered as a regularization of $D = 4$ anomalous chiral gauge theories. The restoration of $D = 4$ gauge invariance should come from the fact that the excitations of heavy fermions living in the bulk must be essential and cannot be discarded, in analogy with ref. [30]. However, the study of the question of how this happens exactly, goes beyond the scope of this paper.

6. The mass gaps

As we have already discussed, a theory of fermionic and gauge zero modes, localised on a string, can be a "good" four-dimensional effective field theory provided bulk modes interact weakly with the localised modes at small energies. A complete analysis of this problem is complicated, and, to our knowledge has never been done for any type of brane-world scenario. Our aim in this section is more modest: we are going to demonstrate that the bulk gauge modes are not separated from zero modes by a mass gap. At the same time, the charged bulk fermions are massive, with a mass gap $m_F \sim 1/a$. This means that at small energies, $E < 1/a$ our theory is indeed the four-dimensional QED. In particular, the processes with visible electric charge nonconservation have a threshhold behaviour, in contrast with the model of ref. [12].

6.1 Gauge fields

To define the spectrum of gauge modes one can use general equations of [18] for spin-1 fluctuations. In our case, three vector fields are present - two coming from the metric, $h_{\mu\theta}$ and $h_{\mu r}$, and the third is the U(1) gauge field making the Nielsen-Olesen string. Our interest is related to the bulk excitations with photon quantum numbers. The field, corresponding to photon, decouples from all other vector fields and has the form of (2.10), where W_μ is now a function of r and x^μ (but not of θ). Since our metric is regular at $r \to 0$, to understand the structure of the spectrum it is sufficient to consider the $r \to \infty$ limit of the corresponding equation for mass eigenvalues. In what follows, we will assume that $M_W > c$, so that $P(r)$ can be neglected in comparison with e^A, e^B at large distances. Then the equation defining the spectrum of bulk photons with four-dimensional masses m_w follows directly from eq.(42) of [18] and is, at $r \to \infty$:

$$-e^{-\frac{3}{2}B} \frac{\partial}{\partial r} \left[e^{\frac{3}{2}B + A} \frac{\partial W(r)}{\partial r} \right] = m_w^2 W(r)\,. \tag{6.1}$$

At large r the asymptotic of the solution is simply a collection of plane waves

$$W(r) \to z^2 \left(C_1 \sin(z) + C_2 \cos(z)\right) , \tag{6.2}$$

where $z = \frac{m_w}{c} e^{cr}$, what proves the absence of a mass gap.

6.2 Fermions

A similar consideration can be carried out for fermions. For large r, the Yukawa term and Majorana mass in (3.1) can be neglected, and the equations for bulk modes (3.7) reads (we consider only 4_+ six-dimensional fermion, the case of 4_- is treated in full analogy):

$$\begin{aligned} \mathrm{e}^{+\mathrm{i}\theta} \left(\partial_r + \frac{\mathrm{i}}{a}\mathrm{e}^{-B/2}\partial_\theta\right) \psi_L &= -e^{-\frac{1}{2}A - 2\int^r d\rho e^{-\frac{B}{2}} e_L A_\theta} \gamma^\mu \partial_\mu \psi_R \, , \\ \mathrm{e}^{-\mathrm{i}\theta} \left(\partial_r - \frac{\mathrm{i}}{a}\mathrm{e}^{-B/2}\partial_\theta\right) \psi_R &= e^{-\frac{1}{2}A + 2\int^r d\rho e^{-\frac{B}{2}} e_L A_\theta} \gamma^\mu \partial_\mu \psi_L \, . \end{aligned} \tag{6.3}$$

The ansatz $\psi_L = e^{im\theta}\psi_L^m$, $\psi_R = e^{i(m+1)\theta}\psi_L^m$ removes the angular dependence and leads, for large r, to the following equation for the spectrum:

$$\left[-\frac{\partial^2}{\partial r^2} + \frac{2ne_L/e - 1}{a} e^{-\frac{B}{2}} \frac{\partial}{\partial r} + \frac{m(m+1-2ne_L/e)}{a^2} e^{-B}\right] \psi_L^m = e^{-A} m_F^2 \psi_L^m \, , \tag{6.4}$$

where n is the string winding number and m_F^2 are the eigenvalues of the 4-dimensional mass^2 operator ∂^2. At large r, asymptotics of the solutions are

$$\psi_L^m \to \frac{1}{\sqrt{z}} \exp\left(-\frac{1 - 2ne_L/e}{2am_F} z\right) \left(C_1 \sin(\omega z) + C_2 \cos(\omega z)\right) , \tag{6.5}$$

where $z = \frac{m_F}{c} e^{cr}$ and

$$\omega^2 = 1 - \frac{(m - ne_L/e + \frac{1}{2})^2}{(am_F)^2} \, . \tag{6.6}$$

The numerator on the right hand side of $\omega^2 - 1$, is nothing but the square of the charge operator $Q = i\partial_\theta - \frac{1}{2} + n\frac{e_L}{e}$ acting on ψ_L. It is easy to work out a similar solution for ψ_R and show that the states ψ_R with the same charge as ψ_L can be put together to construct a four component massive charged spinor.

From (6.3) one can see that at small r and $m_F \neq 0$ and for any value of $m > 0$ there is only one regular wave function with the behaviour $\psi_L^m \simeq 2C(m+1)r^m$, $\psi_R^m \simeq -Cm_F r^{m+1}$, where C is an arbitrary constant. This means that the spectrum of the mass operator is discrete for $m_F < \frac{1}{a}Q$ and continuous afterwords, see eq. (6.6). Thus, the masses of the bulk Dirac fermions are bounded by their charge, viz, $m_F > \frac{1}{a}Q$, and the charged zero modes are separated from other modes by a mass gap.

7. Conclusion and outlook

In this paper, starting from an anomaly free U(1) gauge theory in $D = 6$ we have constructed a full fledged effective $D = 4$ electrodynamics of charged particles interacting with photons and gravitons. Due to gravitational interactions in $D = 6$ the theory dynamically localises massless (or massive, if a Majorana-type mass term is added in $D = 6$) charged fermions and photons to a small region around the core of a Nielsen-Olsen vortex.

Of course, there is a very long way from gravitating quantum electrodynamics to a realistic model incorporating non-abelian gauge fields. The U(1) theory we discussed may be considered as a prototype of such a model. It would be interesting to extend our work to anomaly free supersymmetric models in $D = 6$. Such models do exist [31] and they contain Yukawa type couplings between fermions of different chiralities which played an important role in our construction.

Another question to ask is what is the low-energy behaviour of an anomalous $D = 4$ chiral gauge theory derived from an anomaly free theory in $D = 6$. In section 5 we argued that our scheme can be considered as a regularisation of the anomalous $D = 4$ chiral gauge theories. It is clear that by including a finite number of massive fermionic modes, which have vector-like couplings, the theory cannot be made anomaly free. This implies that all the infinite number of fermionic modes must be incorporated, i.e. for the full consistency of the theory the heavy fields should not completely decouple from low energy physics. The study of this question deserves further investigation.

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References

[1] V.A. Rubakov and M.E. Shaposhnikov, *Do we live inside a domain wall?*, *Phys. Lett.* **B 125** (1983) 136.

[2] K. Akama, *An early proposal of 'brane world'*, *Lect. Notes Phys.* **176** (1982) 267–271 [hep-th/0001113].

[3] M. Visser, *An exotic class of Kaluza-Klein models*, *Phys. Lett.* **B 159** (1985) 22 [hep-th/9910093].

[4] G.R. Dvali and M.A. Shifman, *Domain walls in strongly coupled theories*, *Phys. Lett.* **B 396** (1997) 64 [hep-th/9612128].

[5] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315].

[6] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[7] V.A. Rubakov, *Large and infinite extra dimensions: an introduction*, *Phys. Usp.* **44** (2001) 871–893 [hep-ph/0104152].

[8] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[9] S.L. Dubovsky and V.A. Rubakov, *On models of gauge field localization on a brane*, *Int. J. Mod. Phys.* **A 16** (2001) 4331 [hep-th/0105243].

[10] T. Gherghetta and M.E. Shaposhnikov, *Localizing gravity on a string-like defect in six dimensions*, *Phys. Rev. Lett.* **85** (2000) 240 [hep-th/0004014].

[11] I. Oda, *Localization of matters on a string-like defect*, *Phys. Lett.* **B 496** (2000) 113 [hep-th/0006203].

[12] S.L. Dubovsky, V.A. Rubakov and P.G. Tinyakov, *Is the electric charge conserved in brane world?*, *J. High Energy Phys.* **08** (2000) 041 [hep-ph/0007179].

[13] M. Giovannini, H. Meyer and M.E. Shaposhnikov, *Warped compactification on abelian vortex in six dimensions*, *Nucl. Phys.* **B 619** (2001) 615 [hep-th/0104118].

[14] A. Neronov, *Localization of Kaluza-Klein gauge fields on a brane*, *Phys. Rev.* **D 64** (2001) 044018 [hep-th/0102210].

[15] H.B. Nielsen and P. Olesen, *Local field theory of the dual string*, *Nucl. Phys.* **B 57** (1973) 367.

[16] M. Giovannini, *Gauge field localization on abelian vortices in six dimensions*, *Phys. Rev.* **D 66** (2002) 044016 [hep-th/0205139].

[17] M. Giovannini, J.-V. Le Be and S. Riederer, *Zero modes of six-dimensional abelian vortices*, *Class. and Quant. Grav.* **19** (2002) 3357 [hep-th/0205222].

[18] S. Randjbar-Daemi and M. Shaposhnikov, *A formalism to analyze the spectrum of brane world scenarios*, *Nucl. Phys.* **B 645** (2002) 188 [hep-th/0206016].

[19] R. Jackiw and C. Rebbi, *Solitons with fermion number* 1/2, *Phys. Rev.* **D 13** (1976) 3398.

[20] R. Jackiw and P. Rossi, *Zero modes of the vortex-fermion system*, *Nucl. Phys.* **B 190** (1981) 681.

[21] S. Randjbar-Daemi, A. Salam and J. Strathdee, *Spontaneous compactification in six-dimensional Einstein-Maxwell theory*, *Nucl. Phys.* **B 214** (1983) 491.

[22] J. Hughes, J. Liu and J. Polchinski, *Supermembranes*, *Phys. Lett.* **B 180** (1986) 370.

[23] M.V. Libanov and S.V. Troitsky, *Three fermionic generations on a topological defect in extra dimensions*, *Nucl. Phys.* **B 599** (2001) 319 [hep-ph/0011095].

[24] J.M. Frere, M.V. Libanov and S.V. Troitsky, *Three generations on a local vortex in extra dimensions*, *Phys. Lett.* **B 512** (2001) 169 [hep-ph/0012306].

[25] A. Neronov, *Fermion masses and quantum numbers from extra dimensions*, *Phys. Rev.* **D 65** (2002) 044004 [gr-qc/0106092].

[26] S. Randjbar-Daemi and M.E. Shaposhnikov, *Fermion zero-modes on brane-worlds*, *Phys. Lett.* **B 492** (2000) 361 [hep-th/0008079].

[27] S.L. Dubovsky, V.A. Rubakov and P.G. Tinyakov, *Brane world: disappearing massive matter*, *Phys. Rev.* **D 62** (2000) 105011 [hep-th/0006046].

[28] L. Alvarez-Gaumé and E. Witten, *Gravitational anomalies*, *Nucl. Phys.* **B 234** (1984) 269.

[29] L.D. Faddeev and S.L. Shatashvili, *Realization of the schwinger term in the gauss law and the possibility of correct quantization of a theory with anomalies*, *Phys. Lett.* **B 167** (1986) 225.

[30] E. D'Hoker and E. Farhi, *Decoupling a fermion in the standard electroweak theory*, *Nucl. Phys.* **B 248** (1984) 77.

[31] S. Randjbar-Daemi, A. Salam, E. Sezgin and J. Strathdee, *An anomaly free model in six-dimensions*, *Phys. Lett.* **B 151** (1985) 351.